High-order aspherics:
The SMS nonimaging design method applied to imaging optics

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ABSTRACT

The simultaneous multiple surface (SMS) method has been used to design nonimaging devices, such as solar concentrators and collimators, which work near the thermodynamic limit at highest efficiencies. The very high compactness of these devices is obtained through the simultaneous design of two high-order (above 30^{th}) aspheric surfaces. In imaging optics, low-order aspheric surfaces were introduced to correct Seidel aberrations. The ease with which the SMS method calculates higher-order aspheric surfaces offers great advantages in imaging design.

The SMS method can design N rotationally-symmetric surfaces that, by definition, form sharp images of N one-parameter subsets of rays. The design strategy consists in finding the best configuration of these subsets of rays in phase-space, one that ensures that image-quality specifications will be met by all non-design rays. As a first example of an SMS imaging device, a new video projection optics system is presented, featuring extremely short throw distance, high compactness and wide angle projection.

1. INTRODUCTION

The design of imaging optics is a centuries-old art with literally thousands of inventions patented. Nearly all of these are combinations of flats and spherical surfaces, due to their relative ease of manufacture [1]. Surfaces with substantial departure from sphericity are known as aspherics, but aside from known quartic shapes such as the ellipsoid and the paraboloid there was little use of aspheres until the twentieth century, when advances both in theory and fabrication technology brought them into prominence. Aspheric surfaces can advantageously substitute for multiple spherical surfaces, resulting in less costly devices, even if a single asphere is more costly than a sphere [2]. With the advent of injection-molded plastic optics and precision molded glass optics, aspheres entered the optical-technology mainstream.

Applications of aspheric optics fall into two categories, imaging and non-imaging. Non-imaging optics is concerned with illumination and the distribution of optical power, with the defining constraint being the behaviour of only the outermost rays (called edge rays) of a flux distribution [3]. Imaging optics, however, is concerned with the spatial modulation of flux, having the goal of reproducing a particular flux distribution (the object) at another location (the image). Nonimaging optical design need only take care of the edge rays, a relatively small portion of all rays, but an imaging system must send all rays to their appropriate destination, as parameterized by the system magnification m. A ray originating at coordinate point (x,y) on the object must arrive at the image coordinate (mx, my). This is known as the image-mapping requirement. In the real world of imaging, each point on the object typically radiates flux in all forward directions (*i.e.*, nearly hemispherically) and practicality demands that a significant percentage of this flux reach the proper image point, with little or none going anywhere else on the image plane, once it enters the optical system aperture. In the world of theory, however, many aspheric design procedures hold only when rays hitting a surface subtend only a small solid angle, near the surface normal, and relatively parallel to the optical axis. This is enables approximate aberration coefficients to be rapidly calculated for aspheric surfaces in design optimization.

Attaining perfect image mapping for every object point would theoretically require an infinite number of surfaces, but with a limited number of surfaces, state-of-the-art techniques define a merit function to evaluate deviations from ideal imaging, in order to minimize loss of image formation over sampled points of the image plane.

A general difficulty with aspherics is that they generally do not have a closed-form solution for ray intersections, unlike the algebraic ease with which intersections are calculated for flats, spheres, and the other quadric surfaces (torus, cylinder, ellipsoid, paraboloid, adn hyperboloid), enabling optimal designs to be derived just with a formula. Aspherics in general, however, generally require a computationally intense iterative search that closes in on the precise intersection

State of the art imaging optics design is done via optimization techniques using a parametric representation of a selected group and type of optical surfaces. A merit function of those parameters is defined and the search for the optimum of the merit function is done by a computer-aided multiple-parameter algorithm. The implementation of this algorithm may be based on different techniques, as damped least-square methods, simulated annealing, genetic algorithms, *etc.* However, the differentiation between a local and the global optimum is not guaranteed, and the optimization's success depends upon the particular mathematical representation chosen to specify the surfaces. Moreover, usually the optimum found is not too far from the initial guess of the designer, so that solutions far from that guess are not accessible in practice. The only cases in prior art where no optimization is done is based on problems stated in terms of ordinary differential equations. This is the case of the single-surface designs to provide axial stigmatism (that is, correction of spherical aberration of all orders), such as Cartesian ovals or Schmidt correctors, and the case of the two aspheric surface aplanatic designs, such as those by Schwarzschild in 1905 for two mirrors [4].

2. SMS METHOD FOR IMAGING OPTICS

The present paper presents a new version of the simultaneous multiple surface (SMS) method [5], which, in contrast to the prior art methods, can directly calculate multiple rotationally symmetric aspheric surfaces, without restriction on their asphericity, and enables designing with not only with meridian rays but also with skew rays. An advantage of designing with skew rays, or a combination of meridian and skew rays, is that the design ray-bundles can be equispaced in the phase space, which confers better control of the imaging quality.

The SMS design procedure involves the simultaneous point-by-point calculation of N rotationally symmetric aspheric surfaces, with the condition that N uniparametric ray bundles (previously selected by the designer at the input side) are perfectly imaged (no ray-aberration for those rays). Each bundle can be described by expressions such as $x=x(t, \xi)$, $y=y(t, \xi)$, $z=z(t, \xi)$, where t is the parameter along the ray trajectory and ξ is the parameter across the bundle. For each value of ξ, this expression defines a straight line (a light ray trajectory).

The design strategy described herein consists in selecting the uniparametric ray bundles at the input such that a proper sampling of the phase space (*i.e.*, spatial-angular space) at the object is produced, expecting that the perfect image quality for those rays will provide sufficient image quality of the remaining rays, by proximity. In general, how the bundle of rays must behave at the output side is not known in advance, but it is determined during the design, as a result of imposing the condition of zero ray-aberration for the selected rays. The selection of the ray bundles for the design is done by the designer to best meet the design criteria. For instance, at the input side, if the object is small compared to the input pupil, the ray bundles are selected such that the each uniparametric ray bundle departs from a different point of the object . In the dual case in which the input pupil is small compared to the object, the ray bundles are selected such that each uniparametric ray bundle points towards a different point of the pupil. These two cases can be referred to as object and pupil discretization, and are just extreme cases of possible spatial-angular selection of the uniparametric bundles. The bundles that are going to be used here constitute examples of input-pupil discretization at the object side (all rays in each ray bundle point to different points of the pupil). For greater clarity in the explanations, the pupil is located at infinity in the figures of the following paragraph (so the system is telecentric at the object side) and thus all the rays belonging to a single bundle are parallel.

3. DESIGN OF TWO SIMULTANEOUS SURFACES WITH TWO BUNDLES OF SKEW RAYS

Let us consider an optical system (see FIG. 1) that has circular symmetry (in cylindrical coordinates ρ, θ, z, where the z axis is the axis of symmetry and the optical axis of the system). The object plane is at z=0 and image plane is at z=z'. The optical system consists essentially of two aspheric surfaces S_1 and S_2 to be calculated, and several intermediate known surfaces. In general, the number of intermediate surfaces may independently be 0, 1, or more.

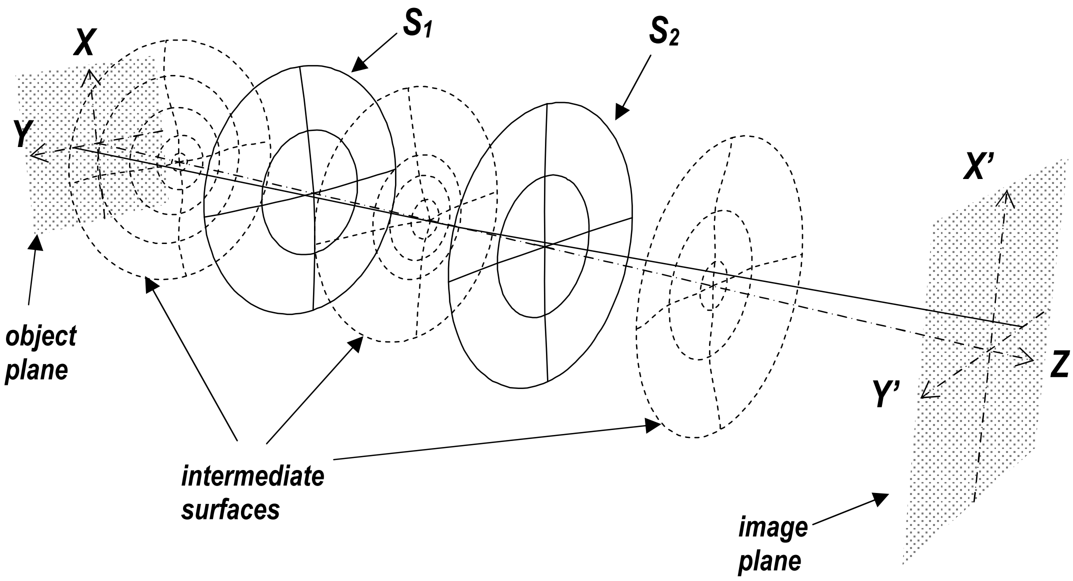

FIG. 1. Layout of a generic optical system with intermediate surfaces between the calculated surfaces and the object and image planes.

FIG. 2 shows the starting conditions of a process for the design of the two optical surfaces S_1, S_2 with two bundles of skew rays in a rotationally symmetric optical system. Uniparametric design bundles N_1, N_2, N'_1 and N'_2 formed by skew rays will be used. The bundles N_1 and N'_1 are respectively symmetrical to N_2 and N'_2 relative to XZ plane. Bundles N_1 and N_2 cross object plane and are given while N'_1 and N'_2 cross image plane and are initially unknown.

In this case, rays (except ray $\nu_{1,0}$) are not contained in a meridian plane. Rays that belong to bundle N_1 are parallel to plane XZ and exit from points (ζ, y_0, 0), where ζ is the bundle parameter, and y_0 is a real number. Thus Cartesian coordinates will be used instead of cylindrical. The object plane is at z=0 and the image plane is at z=z'. Designate the two surfaces to be calculated as S_1 and S_2. Points Q(x, y, 0) on the object plane and points Q'(x', y', z') on the image plane scale as $x' = m\cdot x$, $y' = m\cdot y$, where m is the magnification of the optical system.

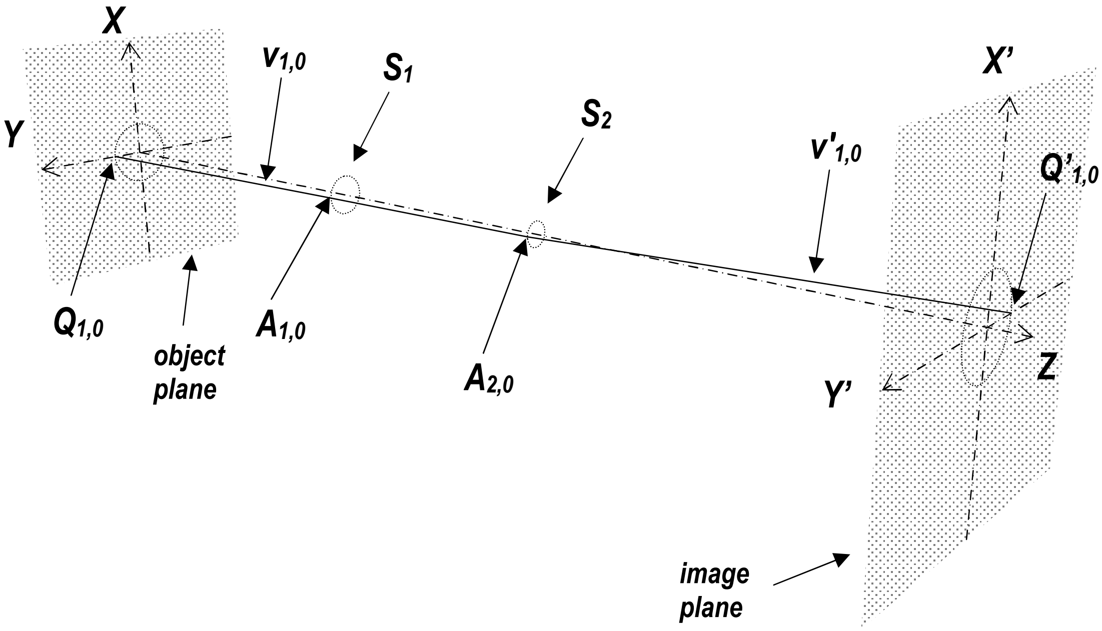

FIG. 2. Initial seeds for the calculation of two surfaces with two bundles of skew rays.

The calculation of surfaces S_1 and S_2 starts with two initial seed points. Define seed point $A_{1,0}$ as a point on surface S_1 at a defined distance from the axis with a known normal vector to surface S_1. Define $\nu_{1,0}$ as the meridian ray of bundle N_1 emitted from a point $Q_{1,0}$ (0, $y_{1,0}$, 0) on the object plane and crossing surface S_1 at point $A_{1,0}$. Calculate the deflection of ray $\nu_{1,0}$ at point $A_{1,0}$ using the known normal vector. Identify the point $A_{2,0}$ as the point on surface S_2 along the path of $\nu_{1,0}$ after it is deflected at $A_{1,0}$. Alternatively, choose the point $A_{2,0}$ as a second seed point, and calculate the normal at $A_{1,0}$ so that the ray $\nu_{1,0}$ is deflected to $A_{2,0}$. The normal vector to surface S_2 at $A_{2,0}$ is calculated such that the ray deflected at $A_{2,0}$ will intersect the image plane at a given point $Q'_{1,0}$ related to $Q_{1,0}$ by the abovementioned magnification m.

The calculation of the first portions $S_{1,1}$ and $S_{1,2}$ of surfaces S_1 and S_2 is shown in ***FIG. 3***. Consider an incremental offset δx parallel to the x-axis relative to the previously calculated point $Q_{1,0}$ (0, $y_{1,0}$, 0), defining a new point $Q_{1,1}$ (δx, $y_{1,0}$, 0). The correspondent image point is $Q'_{1,1}$ (m δx, m $y_{1,0}$, 0). Due to the system's rotational symmetry, rays $\nu_{1,1}$ and $\nu_{1,1}$ have the same skew-invariant. Point $A_{1,1}$ (placed along the ray belonging to bundle N_1 emitted from point $Q_{1,1}$) and point $A_{1,2}$ (placed along the ray belonging to bundle N'_1 that arrived at point $Q'_{1,1}$) will be calculated at the same time by the resolution of a system of three differential equations: the first equation states the condition upon the tangent of surface S_1 that has to be an unitary vector. The second and the third equations give the conditions that first and second surfaces have to be, respectively, consistent to the surface normals at points $A_{1,1}$ and $A_{2,1}$.

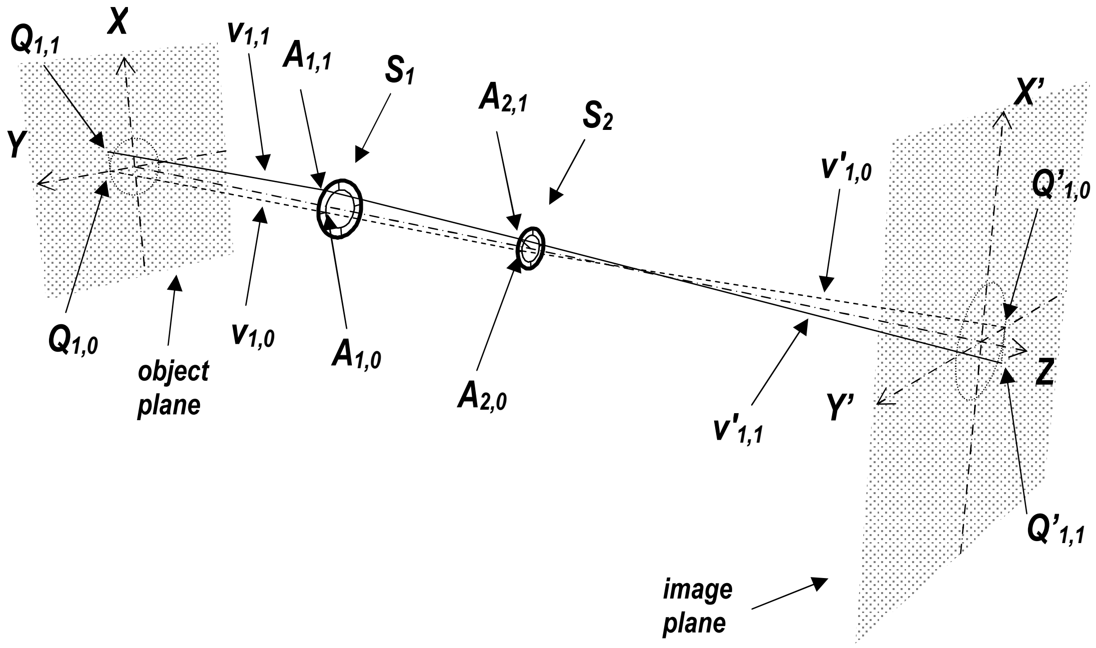

FIG. 3. Calculation of the first points of two surfaces with two bundles of skew rays.

FIG. 4 shows the repetition of the step above using the calculated points $A_{1,1}$ and $A_{2,1}$ and another ray from bundle N_1 to calculate new points $A_{1,2}$ and $A_{2,2}$ and so on, going through all the rays of bundle N_1 that cross the object plane.

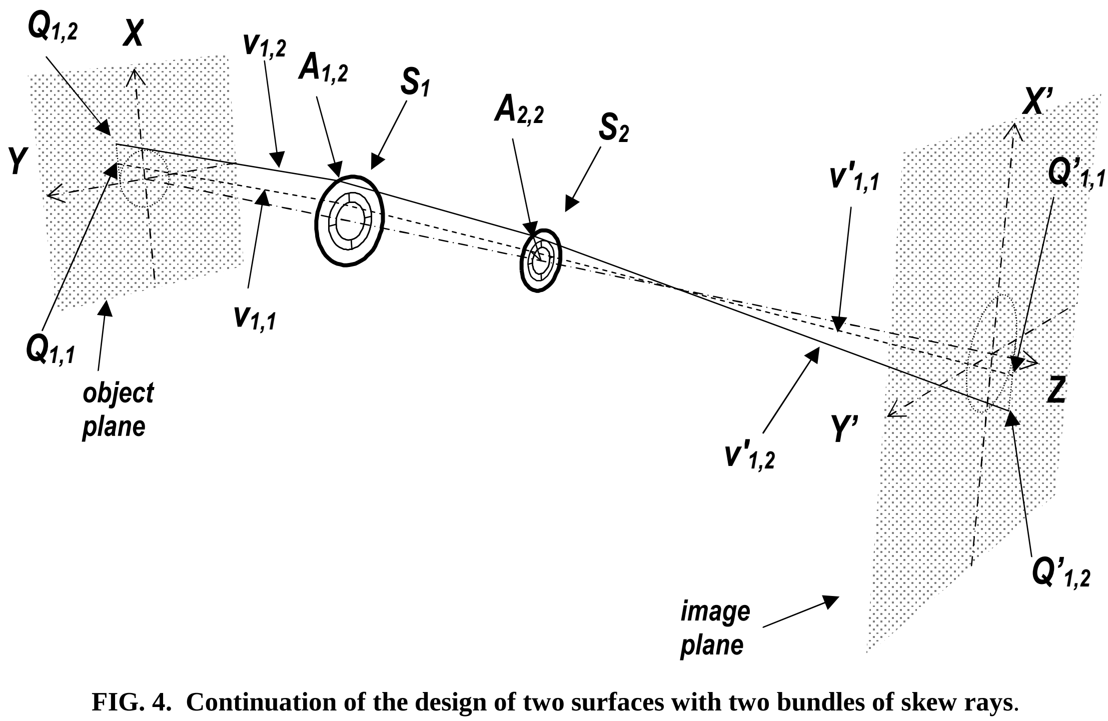

FIG. 4. Continuation of the design of two surfaces with two bundles of skew rays.

It seems that with one design bundle (*e.g.*, N_1) we can calculate two optical surfaces, but in fact we are using at the same time the other, and symmetric, bundle N_2 because we would have calculated the same portions of surfaces S_1 and S_2 if we had used the other bundle N_2. So hereinafter we can say that with one skew bundle of rays (and implicitly with its symmetrical bundles), we can design two optical surfaces.
In these types of designs there is no need for the existence of a previously known portion of surface as a basis to generate a new optical surface, since the two optical surfaces are generated at the same time. Therefore it only takes the choice of two initial seed points to develop both optical surfaces. These two

points are usually off-axis, and sometimes it is necessary to define a portion of surface between each one of them and the optical axis. These portions of surface may be prefixed by an interpolated surface or generated using skew rays. When using skew rays, the method described for ***FIG. 2*** to **FIG. 4** may be used, but first the surfaces must "grow" towards the optical axis by means of rays that will cross, for example, surface S_1 at a smaller radius from the axis than the initial point $A_{1,0}$.

4. APPLICATION OF THE SMS METHOD IN IMAGING OPTICS: PROJECTION OPTICS

These concepts comprise procedures to design optical devices for imaging applications, particularly wide angle projectors. The following exposition relates generally to image-forming optics and more particularly to external aspheric optical mirrors to be adapted to an existing commercial projector. ***FIG. 5*** shows the concept of how external optics is added to a conventional projector to reduce drastically the distance between a screen and the projector. As an example, the horizontal projection distance can be reduced to 20 to 40 cm for an 80" (2 meter) screen diagonal using this external optics. The angle between the optical axis of the mirrors and a line joining the upper edge of the secondary mirror to the upper edge of the screen can be in the 70-80° range. In general there may be an adjustable tilt between the optical axis of the projector and the axis perpendicular to the screen (which is significant if the projected image is to be rectangular and not trapezoidal).

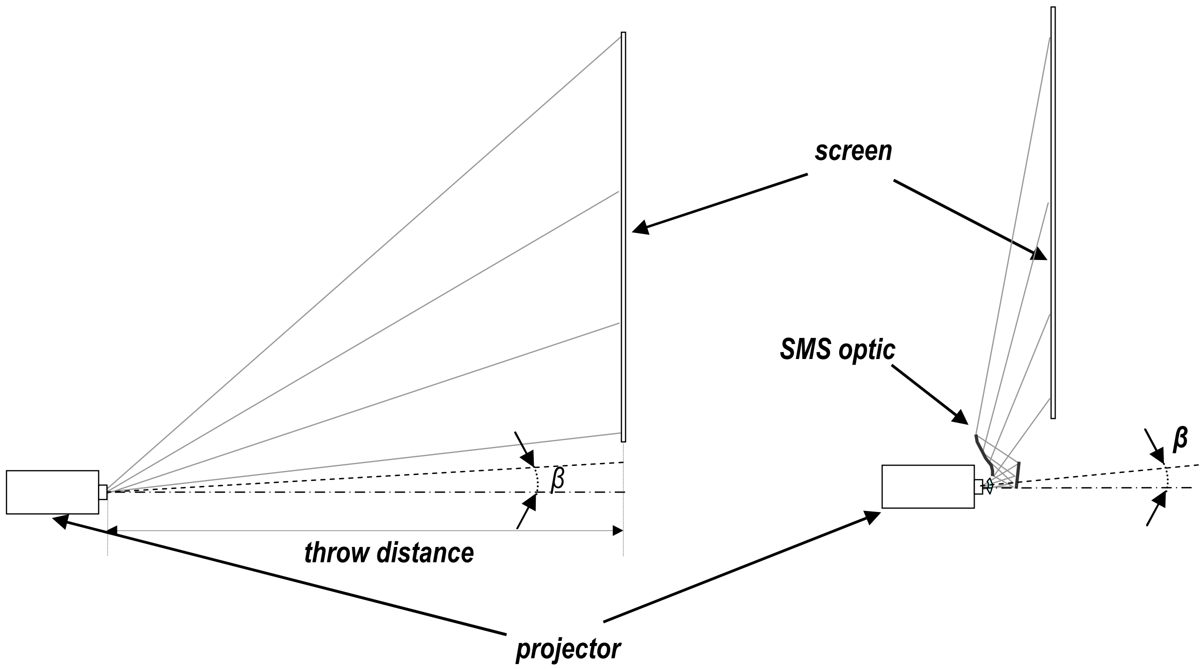

FIG. 5. External SMS optic (right) allows a conventional projector (left)to be nearer to the screen.

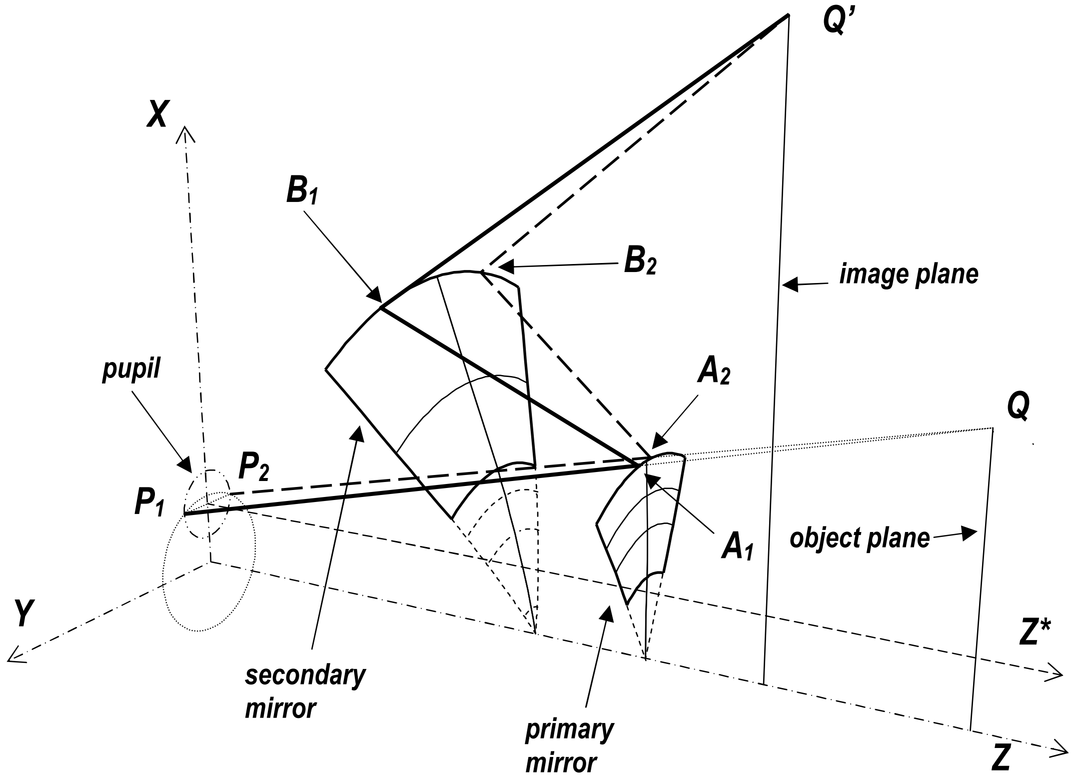

FIG. 6. Design rays for external optic for projectors.

FIG. 6 shows a skew-ray diagram for one application of the SMS method according to ***FIG. 2*** to **FIG. 4** to the design of two simultaneous surfaces (two convex mirrors) with decentred pupil and positive magnification.

The beam from a physical source pupil with edge points at P_1 and P_2 corresponds to projector of ***FIG. 5***, and is initially focused on virtual object with an edge point at Q, corresponding to the conventional screen at a distance from the projector in ***FIG. 5***. (The pupil is decentered with respect to the optical system and the axis that passes through the centre of the physical pupil, denoted as Z^*, is offset respect to the optical axis Z in ***FIG. 6***). The beam is to be redirected onto image surface, with an edge point at Q'. The surfaces of the primary mirror and the secondary mirror are built from skew bundles N_1 and N_2 of rays leaving, respectively, from edge points P_1 and P_2 of the source pupil in the object plane XY.

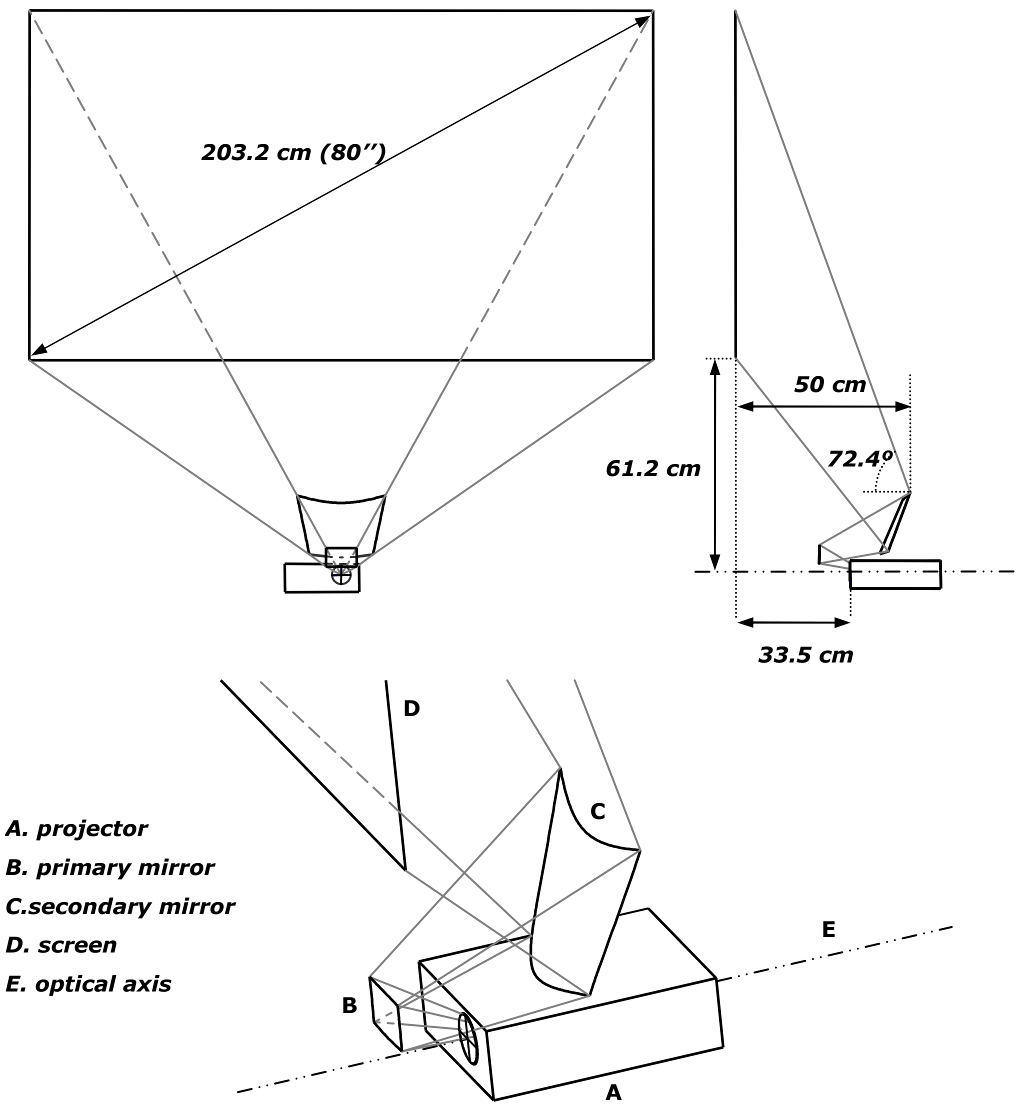

FIG. 7. Frontal, lateral, and perspective views of a two-mirror external optical system for a 80’’ screen.

FIG. 7 shows respectively the front and side view of an optical system for a projector that illuminates a screen at a short throw-distance and the external optics generated as in FIG. 6. The perspective view in

FIG. 7 shows the optical system and projector after the primary mirror and the secondary mirror have been shaped, omitting the parts that are not in the beam's path from the projector pupil to the screen. The primary and secondary mirrors are shown "in air" with the two mirrors' holders are omitted for the sake of clarity, since their construction is completely conventional.

FIG. 8 is a photograph of the prototype of the aforementioned optical system. We have used a conventional BenQ projector and adapted the external optic of

FIG. 7, which has been manufactured by single point diamond turning. Low cost plastic injection moulding is now in progress.

FIG. 8. Prototype of SMS ultra-wide-angle projection mirror with conventional slide projector.

5. CONCLUSIONS

A new version of the simultaneous multiple surface (SMS) method was presented. This method has been used to design optical devices in the field of Imaging Optics. Using two bundles of skew rays, a two-mirror system for ultra-short projection has been designed. The prototype of this system has been also presented.

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LEGAL NOTICE

Method and devices shown in this paper are protected by issued or pending US and International Patents.

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