LED Headlight Architecture that creates a High Quality Beam Pattern independent of LED Shortcomings

Oliver Dross[1a], Aleksandra Cvetkovic[c], Julio Chaves[b], Pablo Benítez[c], Juan C. Miñano[c]
[a]LPI Europe, Marques de Urquijo 14, 5D, 28008 Madrid, Spain
[b]Light Prescriptions Innovators LLC, 16662 Hale Ave., Irvine, CA 92606, US
[c] CEDINT Technical University of Madrid (UPM), ETSI Telecomunicación, C. Universitaria, 28040 Madrid, Spain

ABSTRACT

One of the most challenging applications for high brightness LEDs is in automotive headlights. Optical designs for a low or high beam headlights are plagued by the low flux and luminance of LEDs compared to HID or incandescent sources, by mechanical chip placement tolerances and by color and flux variations between different LEDs. Furthermore the creation of a sharp cutoff is very difficult without baffles or other lossy devices.

We present a novel LED headlight design that addresses all of the above problems by mixing the light of several LEDs first in a tailored light guide called LED combiner, thereby reducing color and flux variations between different LEDs and illuminance and color variations across the LED surfaces. The LED combiner forms a virtual source tailored to the application. The illuminance distribution of this virtual source facilitates the generation of the desired intensity pattern by projecting it into the far field. The projection is accomplished by one refractive and one reflective freeform surface calculated by the 3D SMS method. A high quality intensity pattern shape and a very sharp cutoff are created tolerant to LED to optics misalignment and illuminance variations across the LED surface.

A low and high beam design with more than 75% total optical efficiency (without cover lens) and performance as latest HID headlights have been achieved. Furthermore it is shown that the architecture has similar tolerance requirements as conventional mass produced headlights.

Keywords: SMS Design, Non imaging optics, LED headlight, LED combiner

1. INTRODUCTION

LEDs have been applied successfully in all exterior automotive lighting devices but full headlights. Long life time of LEDs (much longer than the car use time if thermal stress is avoided), compactness, instant switch on, lower power consumption, high color temperature, and, maybe even more important, unique styling possibilities to give cars a high tech look and to distinguish them from competitors, are major drivers for the development of LED front lighting. However, the price of an LED headlight will be much higher than incandescent or even HID solutions and optical, mechanical and thermal hurdles have to be overcome before LEDs will be seen on production car front lights. The cooling of the LED has to be completely conductive as LEDs do not emit energy that is not converted into light as infrared radiation. To solve this problem, new ultra low thermal resistance LED packages on metal core PCB in contact to the lamp housing are utilized. The mechanical design, as will be shown later, is closely connected to the optical performance of an LED headlight as the light of many LEDs has to be bundled with high precision to meet legal specifications and driver comfort. The optical design that will collect the LEDs light and form a legal beam pattern must take mechanical issues as placing tolerances of LEDs into account and it must be efficient to maximize the use of the expensive and still sparse LED flux. Design approaches, developed over many decades for incandescent lighting, fail for LEDs due to their completely different optical behavior.

---

[1] *Odross@LPI-Europe.com; phone +34 915 401 044; fax +34 915 596 082; www.lpi-llc.com

2. THEORETICAL BOUNDARY CONDITIONS OF LED HEADLIGHTS

2.1. LED Flux and number of LEDs for a Headlight

Today's average halogen lamps project about 300 - 400 lm onto the road, very good lamps about 600 lm, while HID headlights reach 900 lm. Conventional incandescent and HID HL use reflectors for the primary collection of the emitted light. Because they cannot use the light from the full solid angle into which the light is emitted, their efficiency is limited. Further losses occur if shutters (projector lamps) are needed to shape the low beam pattern. The optical system efficiency, as defined by the ratio of the flux emitted into the -10/ 10 deg vertical, -40/ 40 deg horizontal far field angle window versus the total emitted flux of the light source, of a conventional system consequently ranges from 30 – 50%. Both incandescent and HID sources produce an abundance of light flux so that less efficient optical designs are tolerable. LEDs, however, with their low flux (and low luminance), call for very efficient optical systems. Because of the smaller solid angle into which an LED emits its light, highly efficient systems are possible. An average system may reach 50%, where specialized systems can reach more than 80% efficiency (as will be shown below).

Optical Efficiency	400 lm on the road	900 lm on the road
50%	800 lm	1800 lm
75%	530 lm	1200 lm

Table 1 LED flux needed for different optical headlight efficiencies and headlight flux values projected on the road.

With today's available flux per white LED (up to 100 lm per package) about 6-15 LEDs have to be employed per headlight function. The higher the number of LEDs, the more difficult is the mechanical and electrical design. A single LED headlight solution may become possible, if the flux per LED package will increase as is has over the last 30 years, sometime between 2008 and 2010.

2.2. Illuminance and Luminance vs. lamp size and LED numbers

The Illuminance (1) is defined as:

$$Ill = \Phi / A_{source}$$

where A_{source} denotes the source surface area and Φ the flux emitted by it. Typical values for a headlight filament bulb is 150 lm / mm^2 and for today's Lumileds Luxeon (2) (at 100 lm and a 1 mm^2 chip= 1.6 mm^2 emitting surface including chip side surfaces) 63 lm / mm^2. It can be expected that within the next 3-5 years the Illuminance of the LED chips will reach the same value as of filaments by improving the LED quantum efficiency and the increasing current density through the LED.

However the luminance of an LED chip compared to a filament is worse because the material used to encapsulate the LED die (needed for protection) has an index of refraction which magnifies the chip:

$$L = \frac{Ill}{n^2 \pi}$$

So a typical automotive tungsten halogen filament displays approximately 48 cd/ mm^2 (n=1 as the filament is surrounded by air) as compared to today's Luxeon LED 8.5 cd/ mm^2 (assuming an index of refraction of the chip encapsulant of n=1.5). As an optical system cannot increase the luminance of the source, the following condition holds:

$$A_{ap} = \frac{I}{tL}$$

where I is the required hotspot intensity of the radiation pattern, t is the transmission efficiency of the system (t describes the transmission losses of the rays in the optical system that travel from the source to the hotspot; as opposed to the optical system efficiency, that is the ratio of the usable flux emitted by the device over the flux emitted by the light source), and L is the LED luminance. A_{ap} is the minimum lit aperture area of the headlight. Assuming a transmission efficiency of 75%, an intensity of 30000 cd for the low beam hotspot, and the above value of LED luminance, a minimum area 48 cm^2 headlight aperture results. This area is the lit area that projects light to the hotspot. The real aperture of a lamp has to be larger because not all of the aperture will be lit, and a portion of the lit area will not emit into the hotspot. Nevertheless a LED lamp can be in the range of today's low beam apertures (typically 110 cm^2). The aperture size calculation shown above is independent of the number of LEDs used for a headlight.

2.3. Etendue conservation and lit aperture size

The LED flux F and the luminance L are related by:

$$\Phi = L \cdot E_{3D}$$

where E_{3D} is the etendue in three dimensions. Because LEDs have low luminance compared to halogen lamps, they must have a large 3D etendue to provide the same flux.

If the optics has planes of symmetry, simple etendue calculations in two dimensions can be applied (for an exception see [11]). The etendue conservation in 2D determines the minimum possible collimation angle in one symmetry plane - the more collimated the light has to be the larger the aperture extension must be and vice versa. Both low and high beam headlight patterns can be viewed as roughly elliptical, with a very narrow vertical extend. Since the LED's have large etendues (also in 2D), the vertical collimation is difficult to obtain. The 2D etendue E for a source that emits ray fans +/- ϑ with respect to its normal from all of its points can be calculated as:

$$E = 2 \cdot d \cdot n \cdot \sin(\vartheta)$$

where d is the line source aperture and n the index of refraction at the source. As the etendue is preserved, it can be seen, that, as one collimates the light (ϑ decreases), the aperture d has to increase. Therefore for a headlight beam pattern a light source aperture that is rectangular or elliptical with greater height then width is preferable. For the Luxeon LED (d=1 mm, n=1.5, ϑ=90deg) the vertical etendue is about 3 mm. If one wants to collimate it's light into a cone of 4 deg of extend (ϑ_e=2deg), which can be taken as a typical vertical extend of a low beam pattern, the lit aperture d_e

$$d_e = \frac{E}{2 \cdot n \cdot \sin(\vartheta_e)}$$

has to be at least 43 mm high. Many types of optics either don't flash the entire height of the exit aperture or increase the etendue. In both cases the real vertical aperture needs to be larger.

The radiation pattern of most non-shielded headlight optics can be understood as a superposition of source images of different sizes and orientations. For most optical systems, in both directions (H, V) the size of those images is controlled by the etendue. However it is possible to create smaller images than the minimum values derived from the source etendue but at the same time the optics will also create larger images: the average angular extent of the images will comply with the etendue conservation.

The full horizontal beam spread is typically 60 deg (40 deg are included between the outermost ECE test points) or 80 deg and more for HID low beam patterns. Therefore an increase of etendue in the horizontal direction of the source is not only tolerable but desirable- a rectangular source much wider than higher would fit nicely into the application.

2.4. Beam pattern formation with LEDs

The legal requirements for a high beam function consist basically of a very high intensity hotspot with an elliptical pattern around it. If sufficient aperture area is available and the optical system efficiency is high, the pattern formation is relatively straight forward as opposed to the low beam functions, both for the SAE regulated marked (North America) and the ECE market [7]. The low beam light pattern emitted by a headlight does not only have to comply with typically 20 legal test points at different angles (with maximum or minimum or both intensity/illuminance values), but also with gradient requirements and car producer requirements on top of legal ones. Furthermore a beam pattern must have a homogenous appearance (no visible "lines" or "holes" in the pattern) and no visible color variations. Roughly a beam pattern (as an example: right hand drive ECE low beam) is composed of (Figure 1):

- a hotspot of high intensity just to the right and below the HV point (driving direction) for far vision
- a wide horizontal "bar-type" pattern below the horizon for road recognition
- a horizontal intensity gradient left of the HV point to avoid blinding oncoming traffic
- a shoulder and elbow to define the hotspot zone
- low glare light values above the horizon to avoid blinding oncoming traffic, but enough light to read overhead signs

The gradient (Figure 2 right) is typically defined at 2.5 deg to the left by searching the highest value G obtained by scanning the intensity at different vertical angles β:

$$G = \log\left(\frac{I_{\beta}}{I_{\beta+0.1^\circ}}\right)$$

The legal minimum of G is 0.08 for SAE and 0.13 for ECE.

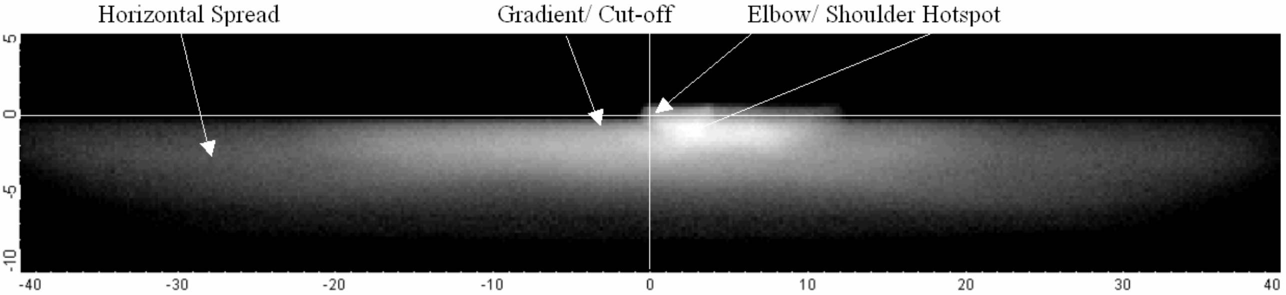

Figure 1: Munsell representation of a typical (hypothetical) low beam pattern as seen shone against a wall at infinite distance. Axis in deg

A conventional headlight design utilizing incandescent or arc (HID) light sources in many cases consist of a single (freeform-) reflector [3]. This architecture fails for LEDs because of the low luminance of the LEDs and their different spatial radiation behavior. In commercially available LEDs the emitters have pronounced variations in luminance. Those from Osram and Cree exemplify high-power LEDs with wires and bonding pads that block light from the top of the emitting chip. In contrast, high-power LEDs from Lumileds exemplify flip-chips, which have no wires or bonds in front. Even in the Luxeon LED luminance varies by a factor of ten from chip center to edge (Figure 1, left), with random patterns in between that differ from one chip to the next. This makes the straight forward approach to image an edge of the LED chip itself to the horizon to form the vertical gradient very difficult. Baffles or other blocking devices to be imaged to form the cut-off drastically reduce system efficiency- this may be acceptable for light sources that produce an abundance of flux- but not for LEDs. Many LEDs also display visible color variations caused by deviations in phosphor layer thickness on the blue chip. Some optical designs transport these color variations into the headlight pattern and thus form unwanted bluish or yellowish zones.

The positioning accuracy of LEDs in production is unlikely to be better than 100 microns. Taking into account that for reasonable lamp exit aperture sizes the projected image size of the chip in the far field will be in average several degrees tall, a movement of the chip of 100 microns shifts the projected source edge by about a 10th of a degree in the far field. As several LEDs have to be used to reach the necessary flux in a beam pattern, the gradient (Figure 1, right) will degrade strongly if the different chip images don’t coincide well. Clearly an optical design is necessary that does neither use LED features to define the gradient and nor images the luminance and color variations of the chip into the pattern.

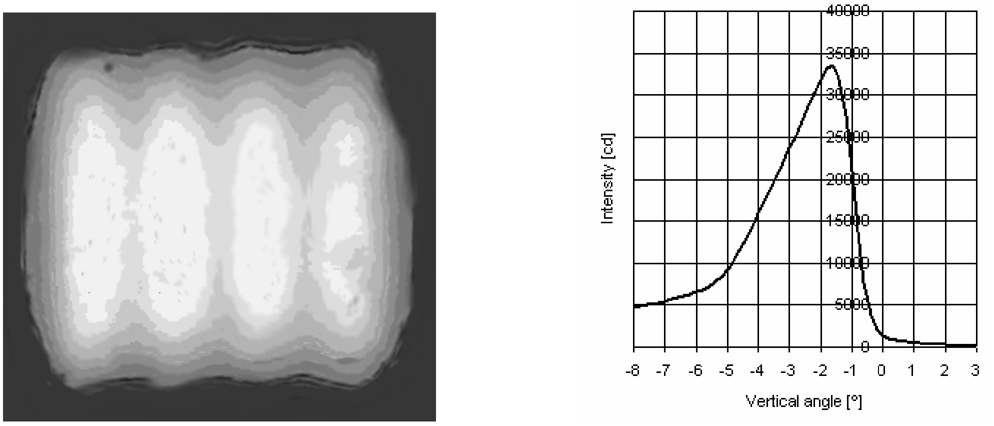

Figure 2 Left: Illuminance pattern of a typical Luxeon chip (1x1mm2). Right: Typical vertical intensity distribution of a low beam incandescent headlight with sharp gradient.

3. THE LED COMBINER XR ARCHITECTURE

Conventional optical designs often use a single free form or faceted mirror to form a headlight beam pattern [3]. The curvature of every mirror point determines the size and position of the image projected from a small surface element, often called a pin-hole. A design method to obtain a single free-form refractive or reflective surface to solve a prescribed-irradiance problem based on the small-source approximation can be found in [4,5,6]. While these solutions work well in the small source limit, large chip LEDs cannot be treated as quasi point sources. Because the SMS [8,9,10,12,13,14] (Simultaneous Multiple Surface) design method explicitly takes into account the extend of the source, it works particularly well for LEDs. The SMS method is the most recent and advanced design tool in non imaging optics. It provides devices that perform very close to the theoretical limits: High collimation and/or prescribed intensity patterns and very small aspect ratios (depth to diameter) are achievable. In the following we present an optical approach that addresses the mentioned problems of LED headlights. The design is partially based on the SMS method.

3.1. Design Concept

In this novel LED headlight concept [15] the light of several LEDs is mixed first to create a secondary source with tailored dimensions and angular spread. This secondary source is transformed by a refractive and a reflective surface into the headlight far field pattern. The elements are:

- Several LED light sources, in this case 3, placed on a the same PCB and heat sink (Figure 3 and Figure 4)
- A tailored light guide called LED combiner that is in optical contact with the LED surfaces. This guide gathers the light from three LEDs with very high efficiency and transforms the luminance distribution of the LEDs into a single prescribed distribution at its exit aperture.
- A refractive free-form lens that is in optical contact with the LED combiner
- A reflective free-form surface

The LED assumed for the shown architecture is not a commercial package. It has a flat exit aperture of about 1.2 x 1.2 mm^2. It consists of a InGaN blue chip with a phosphor coating. If the LED flux is measured in air, the LED combiner will increase the LED output by typically 30-50% because the optical contact to the LED decreases the normal light losses of flat top LEDs that suffer from light confinement and eventually absorption in the LED package due to total internal reflection at it's flat exit surface. The coupling of the LED to the guide is achieved by either optical gel, index matching fluid, or UV curable optical adhesive.

The luminance distribution of LED combiner at its exit aperture facilitates the generation of the desired intensity pattern by projecting it into the far field. The projection is accomplished by one refractive and one reflective free-form surface calculated by the 3D SMS method. The concept is the same for the low and high beam function, although all elements are tailored to each function. A certain number of identical modules consisting of 3 LEDs each and their combined optics are placed as designers desire to form a full low or high beam (Figure 3).

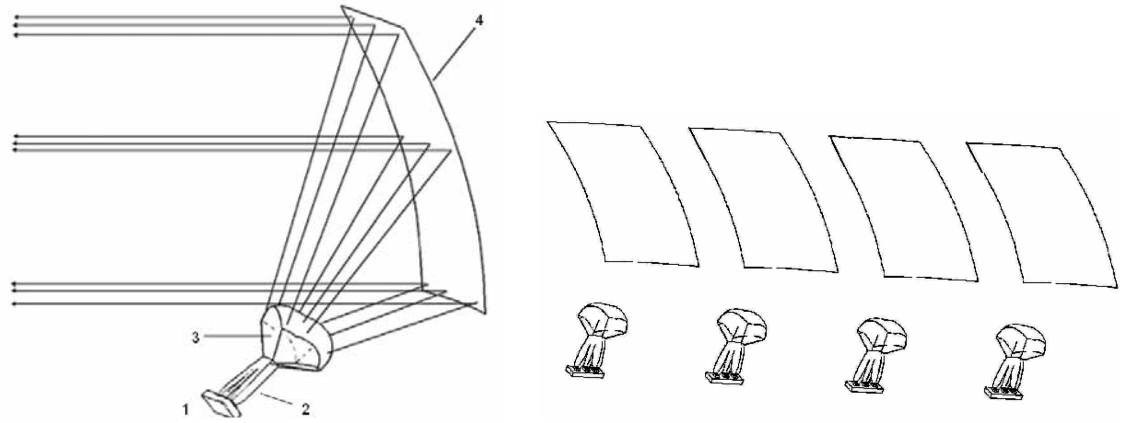

Figure 3: Schematic view of a Combiner RX module (left) and the full LED headlight concept (right). The mirrors can be placed in many different configurations. The combiners can be covered to be invisible for the end user.

3.2. The LED combiner

The LED combiner is a special case of an optical manifold [15]. It is designed as a single plastic injectable dielectric solid, with no metallized surfaces (Figure 4). Its surfaces are tailored to meet the following criteria:

- Attach directly to the LED surface, thereby eliminating the Fresnel loss of the LED surface and increasing the LED efficiency
- Collimate the LED light by the use of total internal reflection
- Combine the light of several LEDs into a single exit aperture while eliminating color and flux variations between different LEDs and illuminance and color variations across each LED surfaces
- Keep the secondary source etendue equal to the etendue of the LED light source to minimize the headlight lit aperture
- Provide sharp features that will be used by the following optical elements to create the gradient and other low beam features

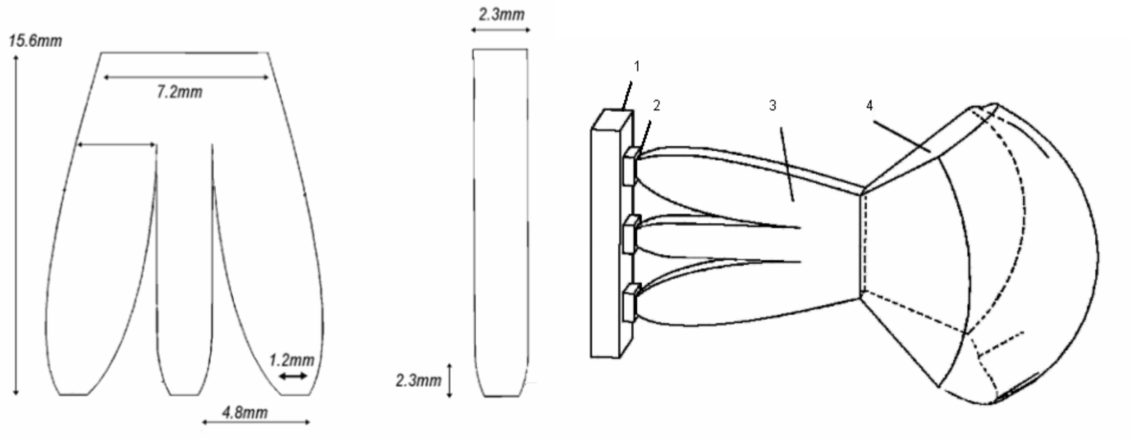

Figure 4: Schematic view of the LED combiner (left and center)) and the source/ combiner/ free-form lens (right)

The luminance distribution at the exit aperture of the combiner is little affected by the usual positional tolerances of the LEDs and their individual illuminance patterns. Its uniformity and definition are superior to other high-luminance light sources such as incandescent filaments or arcs.

3.3. The XR SMS Design

In SMS design method the optical prescription is stated as incoming and outgoing wavefronts. The optical design consists generally of at least two free form surfaces, calculated numerically by the SMS method, that couple the outgoing and incoming wavefronts with each other. The created surfaces can be refractive or reflective. The full optical system may consist of any number of surfaces that exist before, after or in between the SMS surfaces. The effect of these extra surfaces is taken into account by propagating the source and target wavefronts through those surfaces and using the resulting wavefronts as input for the SMS calculation. Besides the obvious physical limitations (i.e. etendue conservation [1]), the only condition for a successful SMS design process is the absence of any caustics of the wavefronts to be coupled in the vicinity of the space to be occupied by the SMS surfaces.

In the XR design the two optical surfaces to be calculated by SMS are a refractive “dome” that is in optical contact with the exit aperture of the LED combiner and a reflective mirror surfaces. The two surfaces will couple two source wavefront WF_{i1} and WF_{i2} with two exit wavefront WF_{o1} and WF_{o2} (see below). Before the 3D SMS calculation can be carried out, a “seed” curve in space can be chosen as a free input parameter. One of the SMS surfaces will start to “grow” from this curve. In the definition of the seed curve lies an important degree of freedom: They can be obtained by a separate simpler 2D SMS calculation using for example WF_{i1}/WF_{o1} and a third pair of wavefronts WF_{i3}/WF_{o3} (Figure 5). The outgoing wavefronts describe, in this example, the vertical extend of the projected source (here the LED combiner exit surface) images. The two straight wavefronts as shown in Figure 5 project one source edge exactly to the horizon (WF_{o1}) and the other source edge to a fixed angle below the horizon. In a real headlight design wavefront WF_{o3} will be curved to create a gradual vertical intensity fall off (compare Figure 2). Although the 3D SMS design does not make use of the third wavefront pair but for the seed curve generation, it maintains the coupling of the third wave front pair in surface regions at least in the vicinity of the seed curve, in most practical cases over the entire design. The 3D SMS calculation has two pairs of wavefronts as input parameters, defined as 3D surfaces and two optical path lengths between the two corresponding pairs of wavefronts. In the shown XR design, two wavefronts are spherical surfaces (WF_{i1} and WF_{i2}) emitted from the two “horizontal” corners of the rectangular exit aperture of the LED combiner light guide. The two corresponding outgoing wavefronts WF_{o1} and WF_{o2} are chosen to prescribe the light emission. In the shown example those wavefront are planar, rotated by a small angle to the left and right from the optical axis respectively. With these wavefronts the optical system would create a rectangular well defined image of fixed angular extend. In the real low beam design, the outgoing wavefronts are derived from the target intensity pattern and are complicated free from surfaces (not shown) that contain detailed information on how the light will be emitted. The SMS calculation now couples WF_{i1} with WF_{o1} and WF_{i2} with WF_{o2}, or, in other words, ensures that all rays emitted from the edges of the light guide will leave the optical device exactly as dictated by the two outgoing wavefronts. Strictly speaking, the SMS design method only controls the light emitted by two, or partially a third point of the source. However, in most practical cases, all other rays emitted by the source from non design points behave “well” in the sense of not showing excessive pin hole image distortion. The calculations are carried out by proprietary software. SMS points generated using the 4 design wavefronts from a point on the seed curve are called chains. The seed curve can be sampled at as many points as desired to create many chains. The full design is eventually defined by all the SMS points that can be interpolated by two 3D surfaces; one of them contains the seed curve.

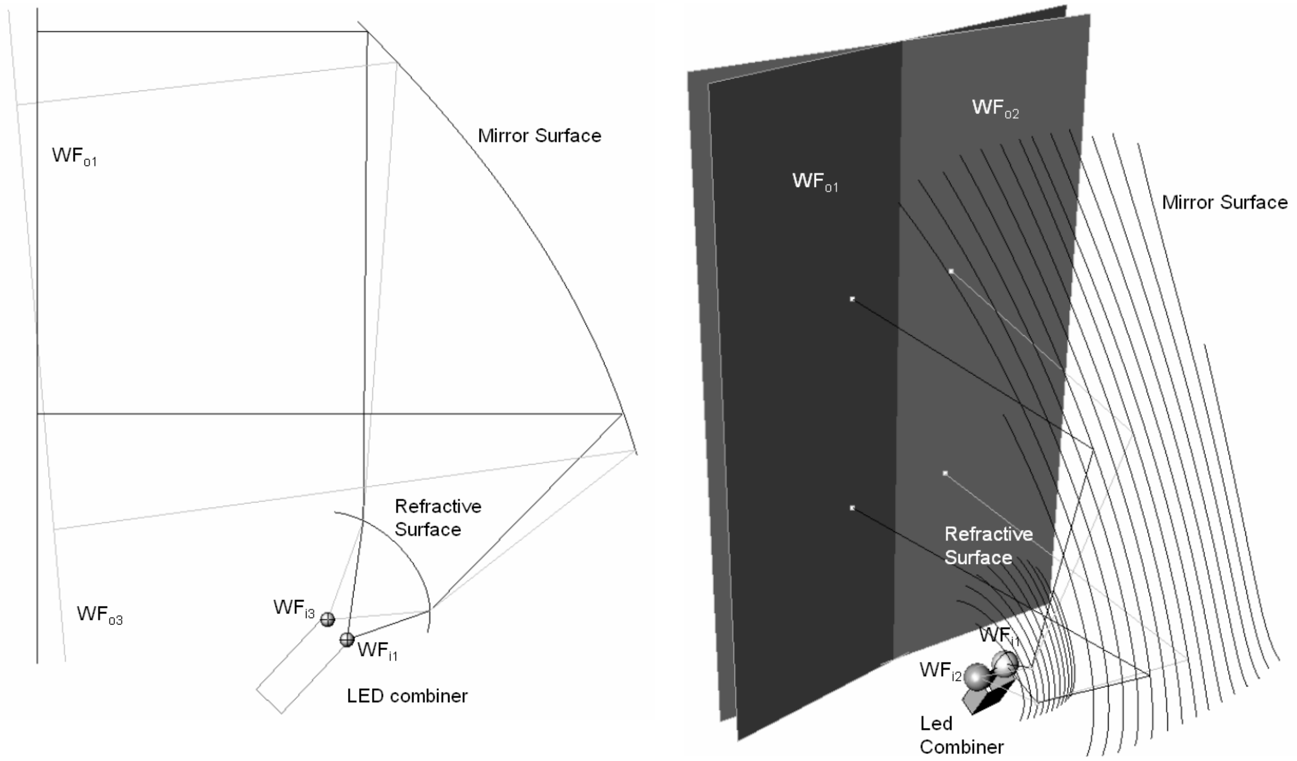

Figure 5: Schematic design procedure for the XR SMS design. Left: 2D design to create seed curve. Right: 3D design that creates SMS “chains” of points in space.

In the case of the low beam, a step-like feature in the light guide exit aperture (not shown) is used to be projected into the far field. Small images are directed to the hotspot and the larger ones to create the pattern spread (Figure 6). The gradient and the step depend on the sharpness of corresponding features of the light guide. Those features are edges of the LED combiner exit aperture. Simple calculations show that the required sharpness of those edges in a mass produced plastic injected LED combiner/ lens is well within standard edge radii.

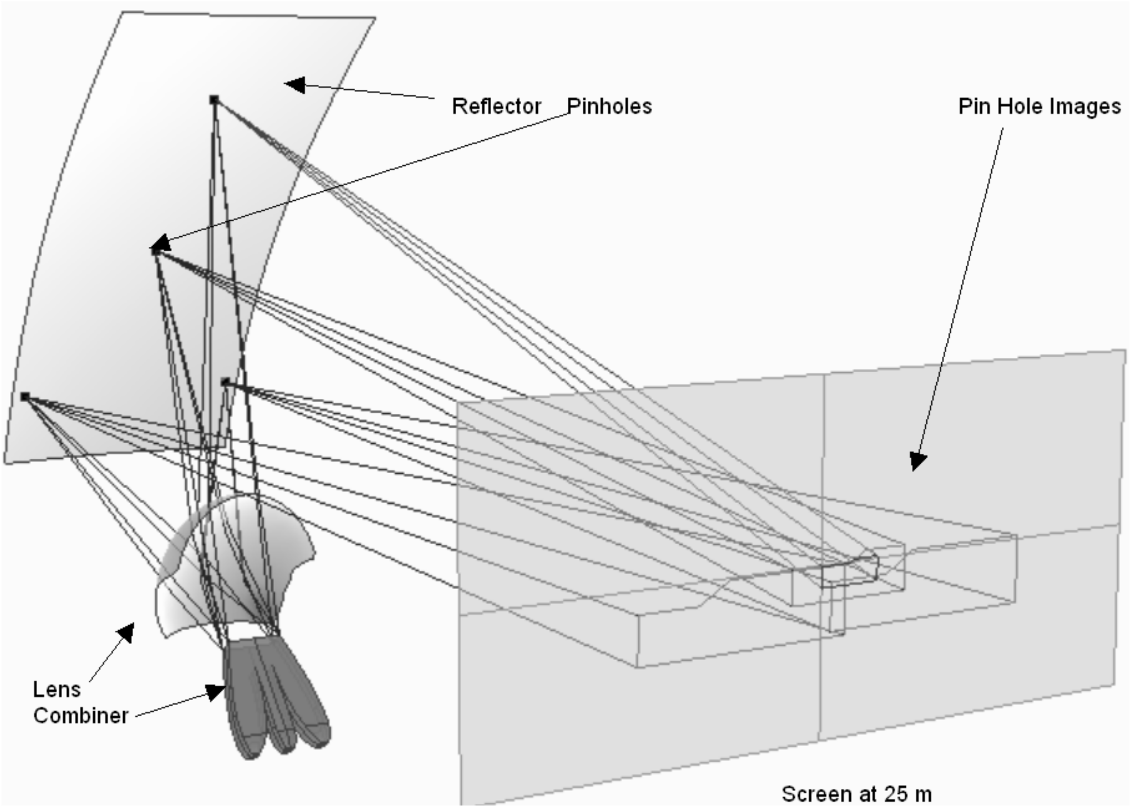

Figure 6: Schematic view of the far field creation with a free-form combiner/ XR. Some pin hole images of different points on the reflector are shown.

3.4. Raytrace Results

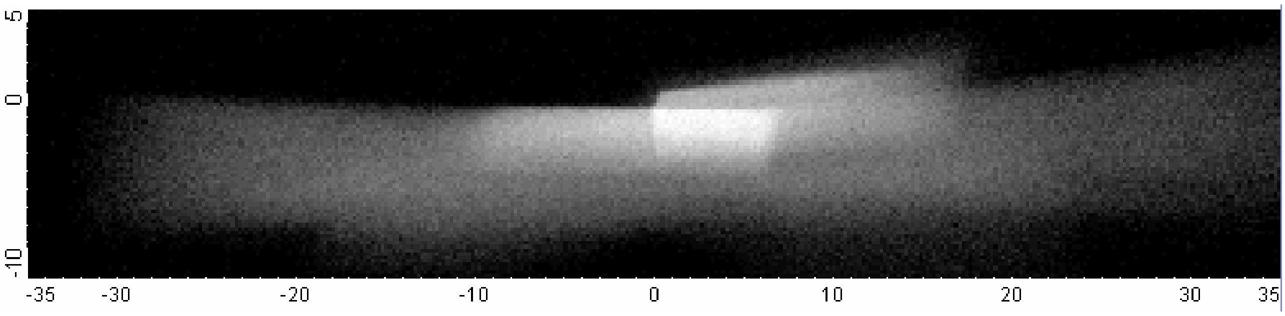

Figure 7: Munsell far field representation of a low beam combiner/ XR pattern.

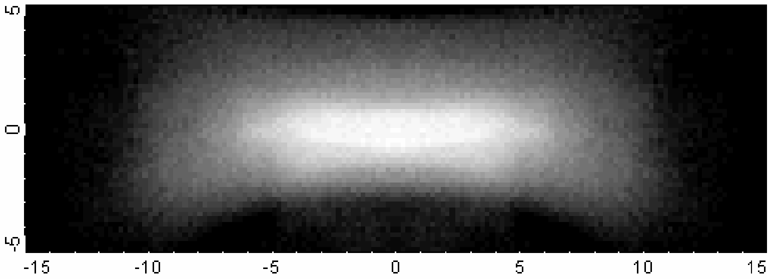

Figure 8: Munsell far field representation of a high beam combiner/ XR pattern.

	Lit aperture size per mirror [mm^2]	Efficiency [%]	Flux on the road [lm]	Hotspot Intensity [kcd]	Full aperture per function [cm^2]
Low Beam	33 x 60	76	860	44	100
High Beam	40 x 74	82	915	82	148

Table 2: Low and High beam performances for XR/ combiner design. All values are derived from raytraces. The efficiency is calculated without a cover glass and bases on the flux projected onto the road compared to the LED emitted flux, assuming the LED had a hemispherical dome attached to it for the flux measurement to avoid light confinement. If the flux of a flat top LED in air was to be used as the efficiency calculation basis, a flux increase of 30-50% is to be expected, raising the total system efficiency to values greater than 100%.

The XR design has been carried out for both a ECE low and high beam. A high quality intensity pattern and a very sharp cutoff are created tolerant to LED to optics misalignment and illuminance variations across the LED surfaces. Detailed raytrace simulations predict extremely high total optical efficiencies (76%/ 82% respectively) and intensities (44000 cd for low and 82000 cd for high beam) that promise exceptional driver viewing.

Both the ECE low and the high beam designs are based on 15 LEDs, 75 lm each (measured with a hemispherical dome attached to the flat LED exit aperture to avoid light confinement within the LED package), 1.2 x 1.2 mm^2 emitting surface, no cover glass and 97% mirror reflectivity (assuming the use of a high reflectivity film instead of conventional Al coating).

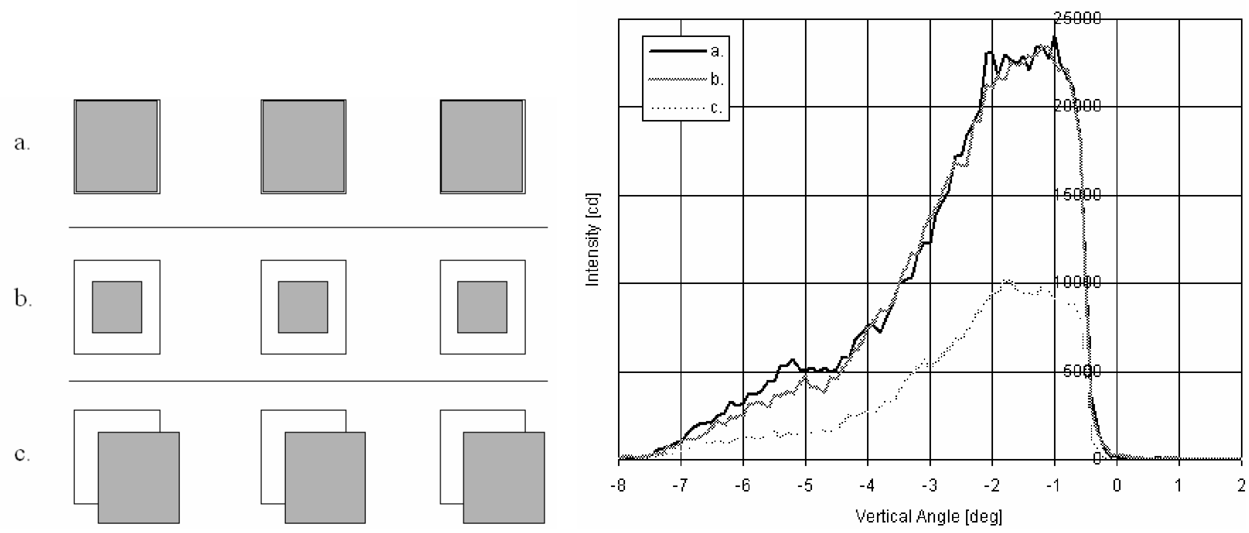

Figure 9 left a: Nominal situation: Entrance apertures of all three arms of the LED combiner are fully filled. b: Test for extreme illuminance variation of LED: All flux is assumed to come from a small center area of 45% of the LED exit area. c: Extreme misalignment case: The LEDs are offset by 400 microns in x and y direction. Right: Gradient position and sharpness for the three cases. Case c shows lower intensities because more than half the light is emitted outside the LED combiner entry surface.

The low beam design has a wide beam spread to mimic HID headlights [Figure 7,Figure 8]. It meets all legal ECE R112 test points. An LED to light guide placing error of up to +/- 0.1 mm has virtually no effect on the light pattern, a 0.2 mm placing error results only in a slight intensity loss. The gradient at 2.5 deg left is calculated to be 0.5, much shaper than the 0.13 legal minimum. In production roughness and impurities of all elements and non cero edge radii of the combiner exit aperture will decrease the gradient values somewhat. Raytraces prove, that the gradient position and value stays completely unaffected by illuminance variations across the LED surface (Figure 9). Even large chip displacements don’t affect the gradient but only reduce the efficiency of the full device as much of the light simply doesn’t enter into the light guide. The shown design used an LED combiner with entrance apertures exactly matched to the LED exit apertures. One may choose the entrance aperture of the lightguide to be slightly larger than the LED so that small LED displacements would have no influence on efficiency and intensity values (i.e. a 1x1 mm^2 source coupled to the 1.2 x 1.2 mm^2 LED combiner surface). The drawback is that such a solution increases the Etendue and therefore reduces the hotspot intensity if the same mirror aperture is maintained.

The low number of modules (5 for 75 lm LEDs) per headlight makes the alignment of theses modules relatively simple. The manufacturing of the free-form mirrors by injection molding and metallization is similar in complexity as conventional headlight mirrors, especially if all mirrors are manufactured as a single part. The shown LED combiner has been sampled by high precision injection molding as a single high quality PMMA element.

4. SUMMARY

The presented LED headlight architecture creates a high quality light distributions and very sharp cutoffs tolerant to LED to optics misalignments and illuminance variations across the LED surfaces. Legal ECE low and high beam designs with >75% / >80% total optical efficiency respectively (without cover lens) and patterns similar to HID headlights have been achieved. Very high hotspots, low glare values, a sharp cutoff and wide beam spread make this headlight an extremely save and comfortable lighting device. The presented design uses a loss free concept to create the gradient and cutoff by imaging features of the optics itself to form the low beam patterns; the design is tolerant to typical LED placing errors and LED illuminance characteristics. Additionally the Combiner/ RX design features color and flux mixing of several LEDs. The design is very compact in terms of aperture and enables stylists to create appealing headlights.

The presented design “LED combiner/ XR” is an example of this architecture that has been developed for the EU project TST3-CT-2003-506316: “Integrated communicating solid-stage light Engine for use in Automotive Forward lighting and information exchange between vehicles and infrastructure”.

REFERENCES

1. W.T. Welford, R. Winston. “High Collection Nonimaging Optics”, Academic Press, New York, 1989
2. Luxeon is a Lumileds high brightness LED, see www.luxeon.com
3. Jung-Hyang Park, Jong-oub Sah, “Design of Reflector Optics with Smooth Surface for Automotive Lamps”, SAE 2001
4. L.A. Cafarelli, V.I. Oliker, “Weak solutions of one inverse problem in geometrical optics”, Preprint, (1994) referenced in L.A. Cafarelli, S.A. Kochengin, V.I. Oliker “On the numerical solution of the problem of reflector design with given far-field scattering data”, Contemporary Mathematics, Vol.226(1999), 13-32
5. S.A. Kochengin, V.I. Oliker, O. von Tempski, “On the design of reflectors with prespecified distribution of virtual sources and intensities”, Inverse problems 14, pp.661-678, (1998)
6. H. Ries, J.A. Muschaweck, “Tailoring freeform lenses for illuminations”, in Novel Optical Systems Design and Optimization IV, Proc. SPIE 4442, pp. 43-50, (2001)
7. For ECE specifications (i.e. Reg 112) seehttp://www.unece.org/trans/main/wp29/wp29regs.html
8. P. Benítez, R. Mohedano, J.C. Miñano. “Design in 3D geometry with the Simultaneous Multiple Surface design method of Nonimaging Optics”, in Nonimaging Optics: Maximum Efficiency Light Transfer V, Roland Winston, Editor, SPIE, Denver (1999).
9. P. Benítez, J.C. Miñano, et al, “Simultaneous multiple surface optical design method in three dimensions”, Opt. Eng, 43(7) 1489-1502, (2004)
10. O. Dross, P. Benitez, J.C. Miñano et al, “Review of SMS Design Methods and Real World Applications”, SPIE, San Diego (2004)
11. P. Benítez, J.C. Miñano, et al, “Étendue squeezing optics: Beating the angle-space compromise of symmetrical systems”, International Nonimaging Optics Workshop 2005, San Diego

Patent list

12. SMS 2D Design method: "High Efficiency Non-Imaging Optics" Patent # 6,639,733, Issued Oct. 28, 2003. Inventors: Juan C. Minano, Pablo Benitez, Juan C. Gonzalez, Waqidi Falicoff and H. John Caulfield.
13. SMS 3D Design method: "Three-Dimensional Simultaneous Multiple-Surface Method and Free-Form Illumination-Optics Designed Therefrom" (pending). Inventors: Pablo Benitez and Juan C. Minano.
14. LED combiner, XR design:“Optical manifold for light-emitting diodes” (international patent pending). Inventors: Julio Cesar Chavez, Waqidi Falicoff, Juan C. Minano, Pablo Benítez, Oliver Dross, William A. Parkyn, JR, Roberto Álvarez